A Hierarchical Model for the Analysis of Intra-individual Variability

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Overview
This poster demonstrates application of Bayesian models in audiology and hearing science for analyses of individual data. These models are an improvement over classical frequentist approaches by incorporating prior information and pooling across subjects for estimation.

Limitations of Frequentist Estimation
Due to greater uncertainty of performance estimates, statistical estimation of treatment effects is more variable with small data sets. The figures below illustrate this variability by contrasting the distributions of a large number of trials (n=100) and small number of trials (n=10).

Simulation of a two-alternative forced-choice experiment is shown below. Each subject completed a different number of trials, illustrating that a small number of trials increases variability of performance estimates. Circles show estimation of proportion correct with the associated classical (i.e., frequentist) 95% confidence interval. Squares show a subject’s true probability of correct response.

Incorporating Prior Information: Non-hierarchical Bayesian Analysis
Two examples of prior assumptions concerning the probability of a correct response in a two-alternative forced-choice task are shown below.

Use of prior information improves estimates of the probability of correct response from measured scores compared to frequentist estimates.

Pooling Information Across Subjects: Hierarchical Bayesian Analysis
Hierarchical Bayesian models provide a principled way to pool information across subjects when estimating the performance of a given subject. This is done by introducing a distribution over the subjects’ own distributions: the result biases each subject’s estimate of performance toward the mean, resulting in more precise estimates.

Across subjects, the width of confidence intervals varies with the number of trials. The confidence interval of zero for subject #2 suggests certainty, a weakness of frequentist estimation. The confidence interval for subject #6 extends below chance probability, an unlikely occurrence in the context of a two-alternative forced-choice experiment.

Limitations:
- Improper prior distributions can result in too much or too little bias toward the group mean, a problem for which solutions have been identified in statistics research
- Complexity of implementation

References:

Applied Example
We applied hierarchical and non-hierarchical Bayesian models to a sub-set of a large data set documenting proportion correct speech recognition in a variety of speech and noise sound field configurations with directional microphones (cf. Galster and Rodemerk, 2013).

Model estimates of the probability distribution of performance show that hierarchical models give more accurate estimates than non-hierarchical models for scores measured from the same number of trials. Black stars indicate cases in which the hierarchical model estimates based on 20 trials were closer to the final (40-trial, non-hierarchical model) estimates than the non-hierarchical model estimates after 20 trials.

Conclusions
The Bayesian hierarchical framework allows for inference that is strengthened by sensible selection of prior distributions and pooling of information across subjects, allowing more accurate estimates of performance in cases where the data at hand are sparse.

References: